Experiment 4

DETERMINATION OF GRAVITY ACCELERATION BY METHOD OF PHYSICAL PENDULUM

Purpose of the experiment: to study tension of the physical pendulum; to determine acceleration of the free fall by method of physical pendulum.

1 EQUIPMENT

1. Physical pendulum.
2. Milimeter scale.
3. Stop-watch.

2 THEORY

2.1 From the law of universal gravitation it follows that on a body lifted above the ground on the height h , the force

$$\frac{\gamma mM}{(R_E + h)^2} = mg, \qquad (2.1)$$

is exerted, where a quantity $\vec{g}$ is free fall acceleration, γ=6,67·10^{-11} m^{3}kg^{-1}s^{-2} (Nm^{2}kg^{-2}) is gravitational constant, m is mass of body, M is mass of Earth (M=5,98·1024 kg), R_E is the radius of Earth. In a vector form the attractive power can be written down as

$$\vec{F} = m\vec{g}; \qquad (2.2)$$

$\vec{F}$ and $\vec{g}$ are directed towards the center of Earth. For a body, that is near to the ground, h<<R_Z (R_Z≅6,37·106 m)

$$g = \gamma \frac{M}{R_3^2}. \qquad (2.3)$$

The value of the free fall acceleration depends on the latitude of a place: on equator it is equal g=9,780 m/s^2, whereas on a pole respective value is g=9,832 m/s^2.

2.2 In this work the value of g is determined in experimental way by method of physical pendulum. Physical pendulum is a body, which oscillates about a horizontal axis hesitates under action of forces, that do not pass through the center of the masses. In the work a rod is used as a physical pendulum (Fig. 3.1).

Sum of kinetic and potential energy of physical pendulum gives the expression

$$E = \frac{I\omega^2}{2} + mgL(1 - cos\,\alpha), \qquad (2.4)$$

where I is a moment of inertia of the pendulum, about the axis of rotation which passes through the end of a rod. The specific expression for I may be found using parallel axis theorem. Other values in above equation have the next meaning: ω stands for angular speed of pendulum, m is mass of pendulum, g denotes acceleration of the free fall close to the surface of Earth, L is distance from the axis of rotation to the center of mass, α is a deflection angle of pendulum from equilibrium position. We choose the position of stable equilibrium of pendulum as an origin for potential energy magnitude. After differentiation Eq. (2.4) with respect to time we have

$$I\omega d\omega + mgL\,sin\,\alpha d\alpha = 0\,. \qquad (2.5)$$

As $d\alpha = \omega dt$, and angular acceleration is equal to $d\omega / dt = d^2\alpha / dt^2$, instead of Eq. (2.5) we have:

$$I\frac{d^2\alpha}{dt^2} + mgL\sin\alpha = 0. \qquad (2.6)$$

Let us divide both sides of equation (2.6) on I, introduce notation

$$\omega_0^{\ 2} = \frac{mgL}{I} \qquad (2.7)$$

and consider the case of small deviations from position of equilibrium ($\sin\alpha \cong \alpha$). Then from Eq. (2.6) we obtain:

$$\frac{d^2\alpha}{dt^2} + \omega_0^{\ 2}\alpha = 0. \qquad (2.8)$$

The solution of equation (2.8) is harmonic oscillation:

$$\alpha = \alpha_0 \cos(\omega_0 t + \varphi), \qquad (2.9)$$

where α_0 is a maximal deviation angle of pendulum from position of equilibrium (amplitude of oscillations) ω_0 is angular frequency, φ is an initial phase (if in the initial moment of time a pendulum was maximally declined from position of equilibrium then $\varphi = 0$).

Period of oscillations is time needed to perform one full oscillation cycle. For the physical pendulum period T is

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgL}}. \qquad (2.10)$$

From the formula (2.10) it is possible to determine the free falling acceleration

$$g = \frac{4\pi}{T^2}\frac{I}{mL}. \qquad (2.11)$$

Now we have determine the period of vibrations of rod and calculate its moment of inertia in order to calculate g.

3 DESCRIPTION OF EXPERIMENTAL APPARATUS

The pendulum used is a rod with mass m and length l. For a rod, the moment of inertia about an axis, that passes through the center of mass, is given by formula:

$$I_0 = \frac{1}{12}Ml^2, \qquad (3.1)$$

and the moment of inertia about an axis, that passes through an upper end, may be found from parallelaxis theorem :

$$I = I_0 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}Ml^2, \qquad (3.2)$$

where we have used that the distance from the axis of rotation to the center of mass is

$$L = \frac{l}{2}. \qquad (3.3)$$

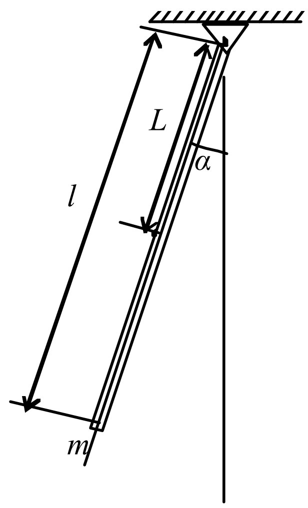

Figure 3.1

Taking into account expressions (3.1)-(3.3), it is possible to determine acceleration of the free falling from equation (2.11) to be

$$g = \frac{8\pi^2}{3}\cdot\frac{l}{T^2}. \qquad (3.4)$$

This there is a computation formula for determination of he free fall acceleration.

4 PROCEDURE AND ANALYSIS

4.1 Determine the length of rod l.
4.2 Determine the period T of oscillations of the physical pendulum. For that, use a stop-watch to measure time t of some number n (specified by teacher) of oscillations and calculate T from formula

$$T = \frac{t}{n}.$$

4.3 Repeat the experiment 3 times. Find the average value of T.
4.4 Calculate the value of the free fall acceleration from equation (3.4).
4.5 Estimate the errors of measurements of length and period and the error of mean value of period calculation.
4.6 Specify the value of relative error $\varepsilon = \left(\frac{\Delta l}{l} + 2\frac{\Delta T}{T}\right) \cdot 100\%$ and absolute error $\Delta g_m = \frac{g_m \cdot \varepsilon}{100}$.
4.7 Express results of the calculation in the form $g = g_m \pm \Delta g_m$.
4.8 Fill the table 4.1 with results of experiments and calculations.

Table 4.1

	l, **m**	***Δl***, **m**	***T***, **s**	***ΔT***, **s**	***g***, **m/s^2**	***Δg***, **m/s^2**	***ε***, **%**
1							
2							
3							
Mean value							

5. CONTROL QUESTIONS

5.1 Formulate the law of universal gravitation.
5.2 What is the physical pendulum?
5.3 Write down an equation of harmonic oscillation.
5.4 Give definitions for amplitude, period, frequency, initial phase of oscillations?
5.5 What assumptions have been made in derivation of formula (2.8)?