Experiment 7

DETERMINATION OF LOGARITHMIC DECREMENT AND DAMPING COEFFICIENT OF OSCILLATIONS

Purpose of of the experiment: to master the basic concepts of theory of oscillations. To determine logarithmic decrement and damping coefficient.

1 EQUIPMENT

1. Pendulum with a scale.
2. Stop-watch.

2 THEORY

2.1 Harmonic oscillations are the variations of physical quantities in time, governed by low of sine or cosine:

$$\alpha = A\cos(\omega_0 t + \varphi_0), \tag{2.1}$$

where α is a value of the varying quantity at the moment of time t, A is an amplitude of oscillations (the maximal value of physical quantity), $(\omega_0 t + \varphi_0)$ is a phase of oscillations (it determines deviation from equilibrium position), φ_0 is an initial phase (it determines deviation at the moment of time t=0), ω_0 is angular frequency.

Period of harmonic oscillations T is the time, required for completion of one full oscillation:

$$T = \frac{2\pi}{\omega_0} = \frac{1}{\nu} \qquad [T]=1\text{c} \tag{2.2}$$

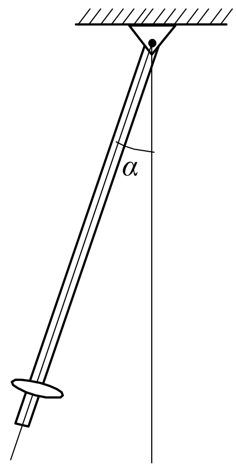

Figure 2.1

ν is frequency of oscillations (number of oscillations per second). Unit of frequency is 1 Hz (cycle per second).

Harmonic oscillations can take place only under action of resilient forces (or other force which return the system to the equilibrium state and is proportional to the deviation from equilibrium $F = -k\alpha$, where α is deviation from equilibrium, k is coefficient of proportionality).

2.2 In real systems friction forces always impede motion. In this work the oscillations are studied on example of physical pendulum (Fig. 2.1). Physical pendulum is a body, that oscillates about a horizontal axis under action of force, that does not passes through the center of the masses of the body.

We may formulate the law of motion of physical pendulum on the basis energy conservation law taking into account that the pendulum rotates about fixed axis (see Fig. 2.1).

Moment of inertia I plays the same role in rotational motion as mass does in translational one. It means that is the moment of inertia is a measure of body's inertia in rotational motion. One can see this from comparison of expression for kinetic energy in rotational motion of a body about a fixed axis ($I\omega^2/2$, where I denotes moment of inertia and ω stands for angular velocity) with expression for kinetic energy in translational motion of a body ($m\vartheta^2/2$).

The moment of inertia of a body of arbitrary geometrical form about some axis can be calculated using following equation

$$I = \sum_i \Delta m_i r_i^2 , \tag{2.3}$$

stating that the moment of inertia of a rigid body about an axis equals to the sum of products of elementary masses (material points) and the square of their distances to the axis. If mass is distributed continuously the sum in previous equation evolves into the integral:

$$I = \int r^2 dm = \int r^2 \rho dV ,$$

where ρ stands for density, dV for elementary volume in the body, r is the distance from the elementary volume dV to the axis of rotation, and the integral is taken over all volume of the body. Unit of moment of inertia is 1 kg·m^2.

One can see that the value of moment of inertia depends on mass of the body, its size, form and the choice of rotation axis. Moment of inertia is additive quantity, that is for the system, that consists of a few bodies, a total moment of inertia equals to the sum of moments of inertia of individual bodies.

A body can rotate around a fixed axis on the condition that there is an external force $\vec{F}$ (or a component of a force), in plane perpendicular to the axis, acting on the body.

The rotational effect of force $\vec{F}$ is characterized by a physical quantity named torque. A torque in rotational motion plays the same role as force does in translational motion. One may calculate torques about an axis or about a point, these differ in general case.

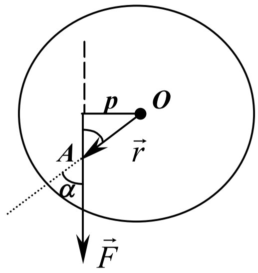

Figure 2.2

We choose the point O on the axis of rotation in the force $\vec{F}$ plane of action (see Fig. 2.2). The magnitude of the torque about fixed axis OZ is the projection of torque $\vec{M}$ about the point O onto an axis OZ:

$$M_Z = Fr\sin\alpha . \tag{2.4}$$

where α is angle between vectors $\vec{r}$ and $\vec{F}$, and $p = r\sin\alpha$ is the length of the perpendicular dropped from the axis of rotation on the line of action of the force, this perpendicular is called lever arm.

If there are several forces acting on a body, resulting (net) torque about a point O equals the sum of component torques.

Unit of torque is N·m.

In the arbitrary moment of time sum of kinetic and potential energies of pendulum equals

$$E = \frac{I\omega^2}{2} + \frac{k\alpha^2}{2} , \tag{2.5}$$

where I is moment of inertia of pendulum about an axis, that passes through the point of suspension ω is angular velocity of pendulum at the given instant, m is mass of pendulum, g is acceleration of the free falling, d is distance from the axis of rotation of pendulum to the center of mass, α is an angle of deviation of pendulum from equilibrium position. Reduction of energy of the system dE during the displacement of pendulum on the corner $d\alpha$, caused by the loss of energy for overcoming the forces of friction

$$dE = dA ,$$

where elementary work of torque of friction forces M_F is

$$dA = -M_F d\alpha , \tag{2.6}$$

the "minus" sign means that the force of friction leads to the decrease of the system energies ($dE<0$).

Suppose that torque of force of friction is proportional to angular speed:

$$M_F = r\omega , \tag{2.7}$$

where r is coefficient characterizing friction, ω is angular velocity ($\omega = \frac{d\alpha}{dt}$). Relation (2.7) comes from the analogy with forces of friction at the motion of material point (for example, mathematical pendulum).

As we have

$$dE = I\omega d\omega + k\alpha d\alpha,$$

then

$$I\omega d\omega + k\alpha d\alpha = -r\omega d\alpha.$$

Forasmuch

$$I\frac{d\omega}{dt} + k\alpha + r\frac{d\alpha}{dt} = 0,$$

or, taking into account, that

$$\frac{d\boldsymbol{\omega}}{dt} = \frac{d^2\boldsymbol{\alpha}}{dt^2}, \quad \frac{d\boldsymbol{\alpha}}{dt} = \boldsymbol{\omega},$$

we have

$$\frac{d^2\alpha}{dt^2} + 2\beta\frac{d\alpha}{dt} + \omega_0^2\alpha = 0, \qquad (2.8)$$

where notations $\beta = \frac{r}{2I}$ and $\omega_0^2 = \frac{k}{I}$ have been introduced. Quantity β is called the damping coefficient of oscillations, ω_0 is angular eigenfrequency of oscillations.

The equation (2.8) has solution in the form:

$$\alpha = A(t)\cos(\omega t + \varphi_0), \qquad (2.9)$$

which describes damped oscillations. Here $A(t)$ is amplitude of damped oscillations:

$$A(t) = A_0\exp(-\beta t), \qquad (2.10)$$

A_0 is initial amplitude (at the moment $t=0$), ω is angular frequency of damped oscillations

$$\omega^2 = \omega_0^2 - \beta^2. \qquad (2.11)$$

The period of damped oscillations is given by formula

$$\mathrm{T} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}. \qquad (2.12)$$

Relations (2.9)-(2.12) take place only at condition $\omega_0^2 - \beta^2 > 0$, if $\beta^2 < \omega_0^2$ then oscillations do not appear because of significant resistance of environment.

3 DERIVATION OF COMPUTATION FORMULA

Let the amplitude of damped oscillations at the moment t_1 be

$$A_1 = A_0 e^{-\beta t_1}, \qquad (3.1)$$

and corresponding amplitude at the moment t_2:

$$A_2 = A_0 e^{-\beta t_2}. \qquad (3.2)$$

Divide (3.1) on (3.2) and get:

$$\frac{A_1}{A_2} = e^{-\beta(t_1 - t_2)} = e^{\beta(t_2 - t_1)} = e^{\beta\Delta t} = e^{\beta n \mathrm{T}}, \qquad (3.3)$$

where n is number of complete oscillations made in time $\Delta t = t_2 - t_1$, $T = \frac{\Delta t}{n}$ is period of these oscillations. Taking logarithm of equation (3.3), we get

$$\ln\frac{A_1}{A_2} = \beta nT \ .$$

Where from

$$\beta = \frac{1}{nT}\ln\frac{A_1}{A_2} = \frac{1}{t}\ln\frac{A_1}{A_2} \ . \qquad (3.4)$$

This equation means that damping coefficient is a quantity, reciprocal to the time, during which amplitude is reduced by factor e ($\ln(A_1/A_2)=\ln(e)=1$).

The logarithmic decrement of the oscillation damping is the quantity

$$\lambda = \beta T = \frac{1}{n}\ln\frac{A_1}{A_2} \ . \qquad (3.5)$$

From a formula (3.5) it follows that logarithmic decrement is a quantity, reciprocal to the number of oscillations, during which amplitude is reduced by the factor e.

4 APPARATUS

The apparatus (Fig. 4.1) consists of physical pendulum, that can oscillate about the fixed axis and the electronic block for measuring the number of oscillations and their total time. The period of oscillation for the pendulum used can be changed by changing the position of the load on the bar. To start experiment one should to deviate the pendulum from position of equilibrium (to measure a deviation, angular degrees are marked on the scale), then clear the readings of the electronic block (pushing the button "CLEAR") and to release the pendulum. A photogate fixes the number of oscillations N and total time of oscillations t. Using these data it is possible to find the period of oscillations $T=t/N$. To stop counting oscillations at some moment (for example, at tenths oscillation), one should push the button of "STOP" during the last oscillation (in the above example, when reading of the electronic block is nine).

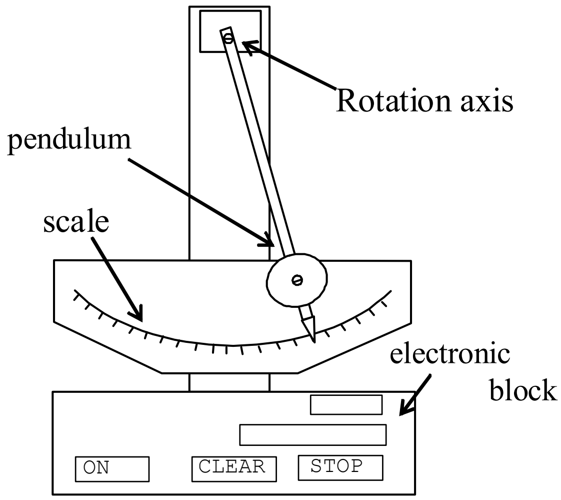

Figure 4.1

5 PROCEDURE AND ANALYSIS

5.1 Deviate the pendulum on angle A_1 (the value of A_1 is specified by lab assistant).

5.2 Set pendulum in motion. Determine the total time of N oscillations (N is specified by lab assistant) and final amplitude A_2.

5.3 Determine the damping coefficient and logarithmic decrement by formulas (3.4) and (3.5), respectively.

5.4 Carry out the experience three times.

5.5 Calculate the average values β_c and λ_c.

5.6 Estimate the errors of measurements and calculations.

5.1 Express results of the calculation in the forms $T = T_m \pm \Delta T_m$ and $\lambda = \lambda_c \pm \Delta\lambda_c$ and specify the value of relative error ε.

5.7 Fill the table 5.1 with results of experiments and calculations.

Table 5.1

	n	t, c	Δt, c	A_1, ...°	ΔA_1, ...°	A_2, ...°	ΔA_2, ...°	β, c^{-1}	$\Delta\beta$, c^{-1}	ε_β %	λ	$\Delta\lambda$	ε_λ %
1													
2													
3													
Mean value													

4.1 Use formula (2.10) to find 5-6 additional values of amplitude in an interval [A_1, A_2] evenly placed in time. Plot graph of time dependence of amplitude.

6 CONTROL QUESTIONS

1. What is harmonic oscillation, and its basic characteristics?
2. What oscillations are called damped?
3. How does the amplitude of oscillations depends on time?
4. What is damping coefficient of oscillations?
5. Give definition of logarithmic decrement of damping.