

Experiment 8

STUDY OF BASIC PRINCIPLES OF THERMODYNAMICS. DETERMINATION OF C_p/C_v FRACTION FOR AIR

Objective: to master the basic principles of molecular physics and thermodynamics. To determine C_p/C_v fraction for air experimentally.

1 EQUIPMENT

1. Hermetic container.
2. Manometer.
3. Pump.

2 THEORY

2.1 Ideal gas law.

The relation between gas parameters is known as ideal gas equation of state (ideal gas law). Only those gases we can treat as ideal in which, first, molecules are very small comparatively to volume of the container and, second, one can neglect interaction between molecules at distance. Equation of state for an ideal gas has the form

$$pV = \frac{m}{\mu} RT, \quad (2.1)$$

where p stands for gas pressure on the walls of container (the unit of pressure is pascal, 1 Pa=1 N/m²), V is volume of the container (in cubic meters), m is mass of the gas (in kilograms), μ is molar mass (the mass of one mole of gas, is measured in kilograms per mol), $R=8,31$ J/(mol·K) is the universal gas constant, T is absolute temperature (in kelvins, K).

If the gas pass from one state (p_1, V_1, T_1) to another (p_2, V_2, T_2) at constant temperature ($T_1=T_2$), the process is isothermal, and

$$p_1V_1 = p_2V_2 \quad (pV = \text{const}).$$

If the process take place at constant volume ($V_1=V_2$), then the process is isochoric (isovolumetric), and

$$\frac{p_1}{p_2} = \frac{T_1}{T_2}.$$

If $p_1=p_2$ (isobaric process), then

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}.$$

2.2 Internal energy of an ideal gas

Internal energy is the total energy of all the thermal motions of molecules and their interactions. For one mole of ideal gas one can calculate the internal energy as a product of the mean kinetic energy of molecules $\langle \varepsilon_k \rangle$ and Avogadro's number N_A , and obtain

$$U = \langle \varepsilon_k \rangle N_A;$$

$$\langle \varepsilon_k \rangle = \frac{i}{2} kT,$$

where i is the number of degrees of freedom for a molecule ($i=3$ for one-atomic molecules, $i=5$ for two-atomic one and $i=6$ otherwise).

For arbitrary mass of a gas we have

$$U = \frac{i}{2} \cdot \frac{m}{\mu} RT. \quad (2.2)$$

2.3 The first law of thermodynamics is the law of energy conservation in application to thermal processes. It reads as

$$Q = \Delta U + A, \quad (2.3)$$

where Q is the heat, obtained by the system from outside, ΔU is the change of internal energy of system, A is the work, done by the gas.

In any process the total work done by external forces is

$$A = \int_{V_1}^{V_2} p dV. \quad (2.4)$$

2.4 The heat transferred to the gas in isochoric process can be expressed as

$$Q = \frac{m}{\mu} C_V \Delta T. \quad (2.5)$$

The quantity C_V is an important physical characteristic of a gas, namely a molar specific heat at constant volume

$$C_V = \frac{Q}{\frac{m}{\mu} \Delta T}; \quad (2.6)$$

from the above equation one can see that C_V is determined by heat needed to warm up 1 mole of an ideal gas by 1 kelvin. Unit of molar specific heat is 1 J/(mol·K).

Analogously, in isobaric process, molar specific heat at constant pressure is

$$C_p = \frac{Q}{\frac{m}{\mu} \Delta T}; \quad (2.7)$$

so, the molar specific heat at constant pressure is the heat needed to warm up 1 mole of an ideal gas by 1 kelvin.

For an ideal gas

$$U = \frac{i}{2} \cdot \frac{m}{\mu} RT,$$

or

$$U = \frac{m}{\mu} C_V T,$$

where

$$C_V = \frac{i}{2} R \quad (2.6)$$

is called the molar specific heat at constant volume

$$dU = \frac{m}{\mu} C_V dT.$$

In isobaric process we have

$$Q = \frac{m}{\mu} (C_V + R) \Delta T,$$

where $\Delta T = T_2 - T_1$. And

$$C_p = C_V + R. \quad (2.9)$$

is the molar specific in isobaric process, so as

$$Q = \frac{m}{\mu} C_p \Delta T . \quad (2.10)$$

2.5 Adiabatic process

A thermodynamic process in which the system does not exchange the energy with its surroundings is known as adiabatic process. In adiabatic process heat transfer is absent

$$Q = 0 . \quad (2.11)$$

The first law of thermodynamics for the adiabatic process reads as:

$$dU + pdV = 0 , \quad (2.12)$$

or

$$\frac{m}{\mu} C_v dT + pdV = 0 .$$

There is a relation between pressure and volume in adiabatic process called Poisson equation

$$pV^\gamma = \text{const} \text{ or } (p_1 V_1^\gamma = p_2 V_2^\gamma) , \quad (2.13)$$

where

$$\gamma = \frac{C_p}{C_v} ,$$

is adiabatic exponent.

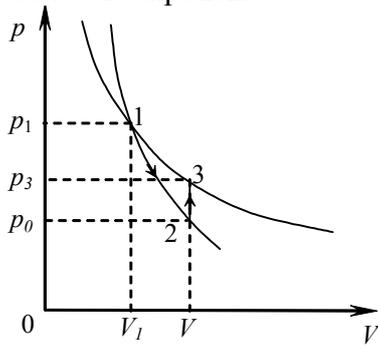


Figure 2.1

In figure 2.1 both adiabatic curve 1-2 and isotherm curve 1-3 are plotted, for comparison. One can see, that in adiabatic process the change in pressure is by much larger than in isothermal process, at the same initial and final volumes. The reason of this is the following: in adiabatic expansion gas perform the work at expense of its internal energy only (no heat is supplied), so the gas temperature quickly decreases. Adiabatic exponent γ can be determined by method of Clemand and Desormes, in which isochoric and adiabatic processes are combined.

3 DESCRIPTION OF EXPERIMENTAL APPARATUS AND COMPUTATION FORMULA

In this experiment we calculate the fraction C_p/C_v by method of Clemand and Desormes.

Container A is connected with manometer M and faucet K. The container can also be connected with the pump to fill it with air.

First, one has to even pressures within and outside the container A, by opening faucet K so that pressure within the container becomes equal to atmospheric pressure p_0 (water levels in manometer M are equal). Then, by pumping air into the container the pressure is increased.

At the same time, temperature increases, so one has to wait for a few minutes to obtain the

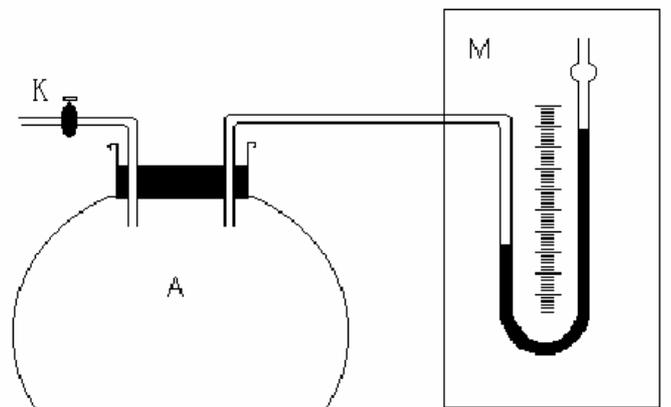


Figure 3.1

thermal equilibrium of the air in container and surrounding (this is a isochoric process, the air volume does not change). After the thermal equilibrium is reached, the difference of levels h_1 in manometer is obtained. In this first state:

$$p_1 = p_0 + \rho g h_1. \quad (3.1)$$

(ρ is the density of liquid in manometer).

If the faucet K is opened and quickly closed again, the air expands adiabatically (segment 1-2 in fig. 2.1), as heat transfer is negligible for such a short process. After that, in the container pressure p_2 equals atmospheric pressure p_0 but temperature decreases $T_2 < T_1$. This is the second state. Transition from the first to the second state is described by Poisson equation:

$$p_1^{\gamma-1} / T_1^{\gamma} = p_0^{\gamma-1} / T_2^{\gamma}. \quad (3.2)$$

When equilibrium state is reached (pressure within the container is equal to atmospheric one) the faucet K is to be closed again. Gas in container is isochorically heated up (segment 2-3 in fig. 2.1) to temperature of the environment T_1 , pressure within the container increases and a difference of liquid levels in manometer appear:

$$p_3 = p_0 + \rho g h_2, \quad (3.3)$$

where h_2 is the difference of levels in manometer. This is the third state of gas with parameters T_1 and p_3 . Transition from the second to the third state is described by equation:

$$p_3 / p_0 = T_1 / T_2. \quad (3.4)$$

Which may be rewritten after taking into account equation (3.1) as:

$$[(p_0 + \rho g h_1) / p_0]^{\gamma-1} = (T_1 / T_2)^{\gamma}, \quad (3.5)$$

or, equivalently

$$(1 + \rho g h_1 / p_0)^{\gamma-1} = [1 + (T_1 - T_2) / T_2]^{\gamma}. \quad (3.6)$$

Combining the former equation with equations (3.3) and (3.4) one obtains

$$h_1 (\gamma - 1) / \gamma = h_2, \quad (3.7)$$

where from we obtain the calculation formula for C_p / C_v

$$\gamma = C_p / C_v = h_1 / (h_1 - h_2). \quad (3.8)$$

4 PROCEDURE AND ANALYSIS

- 4.1 Turn on a faucet K and let the levels of liquid in both legs of manometer M to equalize.
- 4.2 Join the container and the pump and fill the container with air until the difference of levels in manometer reaches 18-20 cm.
- 4.3 Turn off a faucet K and wait until the levels difference does not change (at these instance temperatures inside and outside the container are equal). Determine the difference of levels h_1 in manometer's legs.
- 4.4 For an instance, turn the faucet K to make levels in manometer equal and turn the faucet off quickly.
- 4.5 Wait 2-3 minutes until the air within the container is heated up to the room temperature (the levels in manometer stop changing). Determine the difference of levels h_2 .
- 4.6 Repeat the experiment 4 times more.
- 4.7 Calculate the mean value $\gamma = C_p / C_v$ by formula (3.8).
- 4.8 Estimate the errors of measurements and calculations.
- 4.9 Express results of the calculation in the form $\gamma = \gamma_c \pm \Delta\gamma_c$ and specify the value of relative error ε .
- 4.10 Fill the table 4.1 with results of experiments and calculations .

Table 4.1

	$h_1,$ 10^{-2} m	$h_2,$ 10^{-2} m	γ	$\Delta\gamma$	$\varepsilon,$ %
1					
2					
3					
4					
5					
Mean value					